Wave propagation simulation on regional scales: algorithms and applications

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Abstract

High-performance computers today allow us to approach realistic frequency bands with 3-D numerical simulations. Methods for regular-grid cartesian systems are well established. Numerical algorithms for more complicated structures like spheres, cylinders, models with complex topography, etc. are less evolved. Here we briefly summarize a recently developed algorithm which allows the numerical solution of elastic wave propagation on untructured grids using explicit differential operators. The advantage compared to other techniques for unstructured grids (e.g. finite or spectral elements) lies in the fact that due to the local character of the operators, no matrix inversion is necessary.

We also summarize results from modelling trapped mode wave propagation in fault zones, an area of seismology which - due to the many observations of such waves recently made - is a rapidly expanding field. The question which remains to be resolved is, whether fault zone waves can be used to reliably determine the structure of faults at depth. Only then an assessment of potential ruptures of fault systems would be possible.

Introduction

Large scale modelling of wave propagation requires flexible methods which are well adapted to parallel supercomputers. The recent technical direction of supercomputers with clusters of shared-memory systems (nodes) favors numerical techniques which use as little internode communication as possible. In the context of solving time-dependent partial differential equations it is therefore important to investigate and optimize methods which only make use of near-neighborhood communication. Below we present a method which allows the numerical solution of wave propagation on unstructured grids without the need for large matrix inversion. This makes such an algorithm very flexible, transparent and easy to parallelize.

We also present recent results from using a classical 3-D finite-difference method to simulate wave propagation in fault zones. To understand the behaviour of fault zones knowledge about the structure at depth is crucial. However, the dynamics of faults is predominantly governed by a narrow zone of low velocity at the center which is almost unvisible to seismic waves using tomographic methods. Yet, these low-velocity zones (LVZ) are capable of trapping wave energy as long as the sources are located within this LVZ. 3-D forward modelling of potential fault structures is important to help us understand whether fault zone trapped waves can be used to determine the structure of faults at depth.

Wave propagation on unstructured grids

For many problems the use of numerical methods for regular grids (e.g. finite differences, pseudospectral method) is inappropriate. Examples are models with spherical or cylindrical geometry or with strong topography. Another difficulty arrises when the elastic properties of models vary strongly. Because the grid density has to be adapted to the smallest velocities, parts of the model may be much oversampled. Thus a grid which adapts its density to the velocity model is desirable. This would lead to unstructured grids which can be handled with techniques from computational geometry (e.g. Delaunay triangulation and Voronoi cells). Numerical methods capable of handling arbitrary structures such as finite or spectral elements are more difficult to implement particularly on parallel computers due to the necessity to invert matrix systems. We therefore investigate techniques which allow the explicit calculation of partial derivatives - similar to finite differences - but using a set of unstructured points in the neighbourhood without matrix inversion.
We make use of the concept of natural neighbours introduced to geophysics by Sambridge et al. (1995)[5]. As the common first-order wave equation system suggests the use of a staggered grid, we define two grids, one with velocities, the second with the stress components (Käser et al., 2000)[3]. For each point the natural neighbours defined on the complementary grid have to be found as they are used for the calculation of the partial derivatives. They derivative weights can be calculated using the finite-volume approach or natural neighbour coordinates (Sambridge et al., 1995)[5]. The latter approach uses overlapping Voronoi cells to calculate the weights (Fig. 1a).

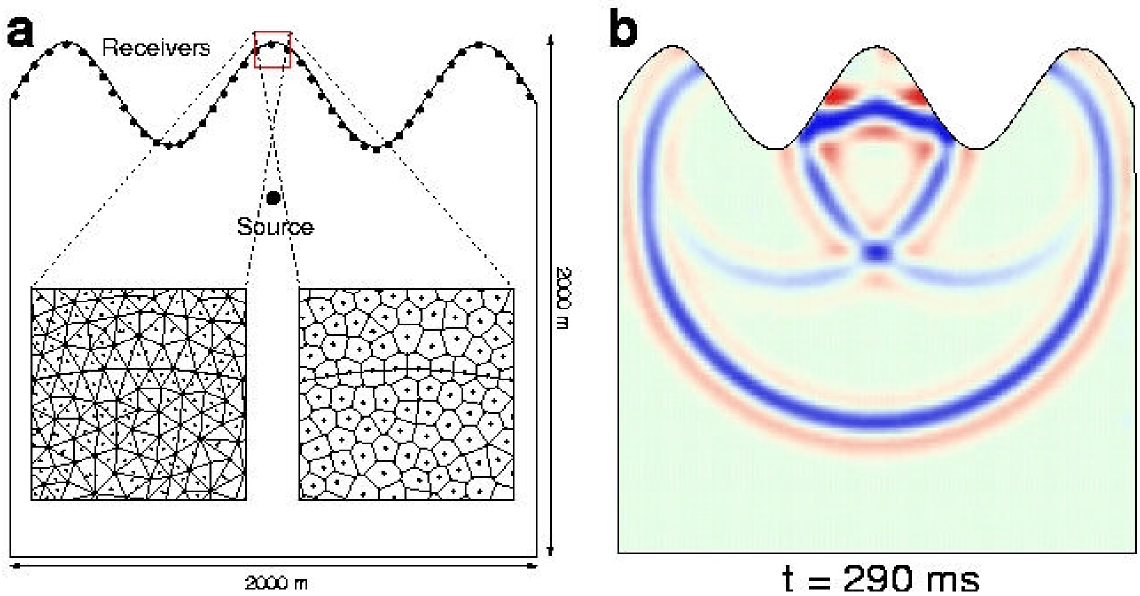

Figure 1: a: Examples of wave propagation on an unstructured grid with topography (acoustic case). The interface is given and the grid defined by a standard grid generator. The inlets show the resulting grids with triangles and the corresponding Voronoi cells. b: snapshot of wave propagation showing the focusing of energy due to topography (Käser and Igel, 2000)[4].

Once the differential weights are calculated the method can be implemented like a finite-difference algorithm. The parallelisation can be carried out using domain decomposition. The accuracy of the differential operators is discussed in Käser et al. (2000)[3] while the method is applied to wave propagation problems in Käser and Igel (2000)[4].

Example: fault zone wave propagation

Fault zone (FZ) structures play dominant roles in fault mechanics and strong ground motion in seismically active regions. Although a huge amount of seismograms recorded in FZ areas exists, the structure of FZ at depth is not well understood. One reason is the low FZ thickness which makes the structure invisible for seismic tomography because the properties of body waves crossing the fault are hardly altered. A better approach to reveal the structure of a FZ is the use of FZ guided head and trapped waves (FZ-waves) which sometimes arise in the fault region and which are caused by low seismic velocities in the FZ. These waves can travel many kilometers inside the fault before reaching the surface and therefore are strongly altered by the FZ properties.
Currently modeling of FZ-waves is mostly done in 2D and is not able to handle realistic sources like double-couple point sources or strong deviations from a 2D geometry of a FZ.

We recently carried out simulations of 3D elastic wave propagation through various FZ models. Our algorithm is based on a high-order staggered-grid finite-difference scheme. The algorithm was carefully checked against analytical solutions presented by Ben-Zion (1990)[1], see Fig. 2.

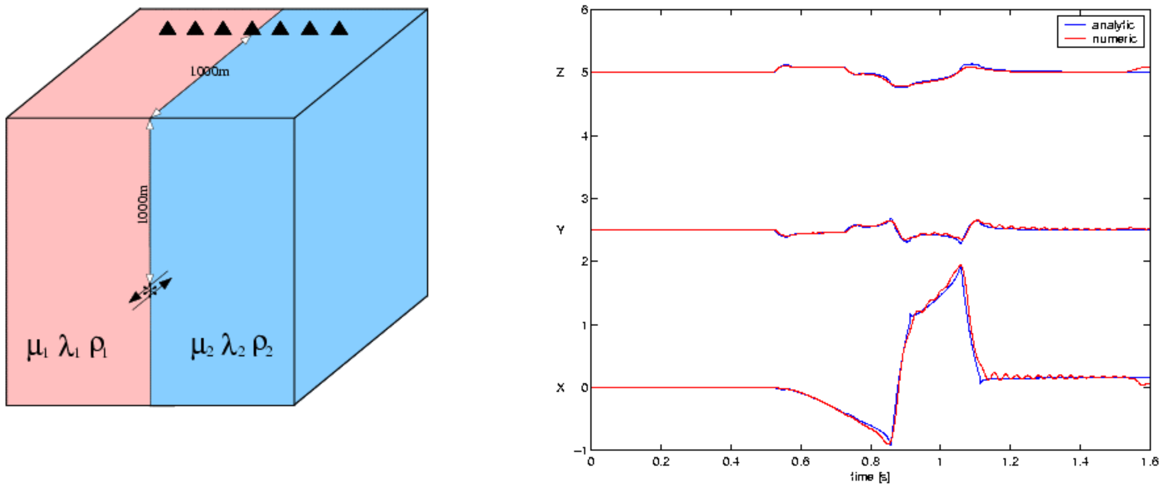

Figure 2: Left: Geometry of the analytical problem. The double-couple source is located at a material interface generating a complex wavefield. Right: Comparison of numerical (unfiltered) and analytical solutions for a source with a ramp-like source time function. The numerical artefacts vanish after filtering.

We discuss the influence of some fault geometries and the seismic properties on seismograms observed at the earth surface, with a focus on the envelope and the frequency content of the seismic traces. Our simulations show that a fault which is continuous at depth and split into two fault segments towards the surface is capable of guiding FZ-waves. Even for a source which is located beneath one of the segments clear FZ-waves can be measured also on the other fault segment. Therefore the detection of FZ-waves on different fault segments can be used to find out whether the segments are continuous at depth or not. Fault models with an increasing lateral disruption at depth lead to a decrease of FZ-wave amplitude and to a spatial smearing at the surface. If the lateral shift of the FZ exceeds its own width, no more distinct FZ-waves arrive at the surface. The spectra for models with increasing amounts of lateral disruption look similar, whereas the FZ-wave amplitude decreases and the wave train becomes longer. Thus the appearance of FZ-waves and their properties contain information on how continuous the fault is at depth. The effect of varying fault width, e.g. a bottleneck structure, on the amplitudes is moderate. Therefore moderate changes of the fault's properties with depth hardly alter FZ-waves; such features should be difficult to resolve from seismograms. The effect of a vertical gradient of the seismic properties show that the FZ-wave amplitude, the spectrum and the length of the wave train are unaffected by realistic levels of the gradient. Thus it seems to be difficult to derive information about a seismic gradient from seismograms (see Fig. 3).

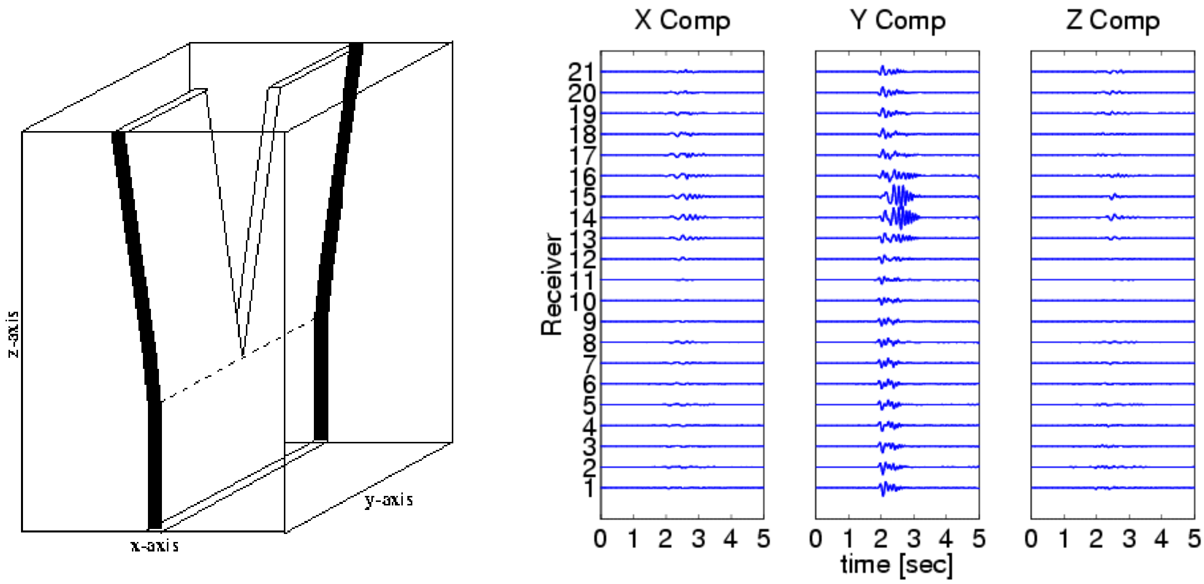

Figure 3: Left: Example of a model with disrupted fault for which synthetic seismograms were calculated by Jahnke et al. (2000)[2]. Right: Synthetic seismograms for a model with a vertical fault with a lateral disruption at depth. On the y-component the trapped mode energy can be identified.

Acknowledgments

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